Exercise 1 (3 pts)
LN Exercise 1.32

Exercise 2 (4 pts)
LN Exercise 2.21.

Exercise 3
Let $X_1, X_2, \ldots$, be i.i.d. integrable random variables with values in $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$, and defined on the probability space $(\Omega, \mathcal{F}, P)$. Further, assume that $EX_n = 0$, and $\alpha = EX_n^2 < \infty$, $n \geq 1$.
Denote $M_n = \sum_{k=1}^{n} X_k$, and $M'_n = M_n^2 - n\alpha$, $n \geq 1$. Then, $(M_n)_{n \geq 1}$ and $(M'_n)_{n \geq 1}$ are both martingales, adapted to $(\mathcal{F}_n^X = \sigma(X_1, \ldots, X_n))_{n \geq 1}$.

Let $\tau$ be an $(\mathcal{F}_n)_{n}$-adapted stopping time with $E\tau < \infty$.

a) (2 pts) Show that $(M''_\tau = M_{\tau \wedge n})_n$ is bounded in $L^2$ (i.e. $\sup_n E(M''_\tau)^2 < \infty$), with $EM''_\tau = 0$, $n \geq 1$. Show that $EM_\tau = 0$.

b) (2 pts) Show that $EM''_\tau = \alpha E\tau$. 